

Spindown of massive rotating stars

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ABSTRACT

Models of rapidly rotating massive stars at low metallicities show significantly different evolution and higher metal yields compared to non-rotating stars. We estimate the spin-down time-scale of rapidly rotating non-convective stars supporting an $\alpha - \Omega$ dynamo. The magnetic dynamo gives rise to mass loss in a magnetically controlled stellar wind and hence stellar spin down owing to loss of angular momentum. The dynamo is maintained by strong horizontal rotation-driven turbulence which dominates over the Parker instability. We calculate the spin-down time-scale and find that it could be relatively short, a small fraction of the main-sequence lifetime. The spin-down time-scale decreases dramatically for higher surface rotations suggesting that rapid rotators may only exhibit such high surface velocities for a short time, only a small fraction of their main-sequence lifetime.

Key words: stars:evolution, stars:general, stars:mass-loss, stars:rotation

1 INTRODUCTION

Models of rotating stars, particularly those of low-metallicity, have generated a great deal of interest because it is believed that their nucleosynthetic output could be significantly affected and so may better fit the observed composition of halo stars and globular clusters. Otherwise unaccountable nitrogen in very metal-poor stars could be explained by rapidly rotating massive stars because their nitrogen yields are found to greatly increase with spin

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(e.g. Meynet et al. 2006). There are suggestions that stars of very low metallicity could have very high equatorial spin velocities, of the order of $600 - 800 \text{ km s}^{-1}$ (Chiappini et al. 2006). Moreover very rapidly rotating stars could induce chemically homogeneous evolution and this may solve a problem of insufficient mass loss at very low metallicity in the collapsar models of γ -ray bursts (Yoon & Langer 2005).

Tout & Pringle (1991) showed how a magnetic dynamo might be driven by rapid rotation and radial turbulence. The mass loss in a magnetically controlled stellar wind results in quite rapid angular momentum loss and hence stellar spin down. In this paper we present an estimate of the spin-down rate for non-convective stars. In all cases the Alfvén radius is larger than the stellar radius so angular momentum may be efficiently removed from the system and this leads to short spin-down time-scales.

Recently, Dervişoğlu, Tout & İbanoğlu (2010) considered the spin angular momentum evolution of the accreting components of Algol-type binary stars and demonstrated how accretion from a disc ought to spin the accreting star up to breakup velocity very quickly, after only a small amount of mass transfer. However, the accreting stars in Algols, which are hot main-sequence stars with radiative envelopes are not spinning at anywhere near their breakup rate. They showed that tides alone are insufficient to explain the spin down and argued that the most likely mechanism for the angular momentum loss is a magnetically controlled wind generated by a dynamo which is itself driven by the rapid rotation. This is further evidence that stars cannot continue to spin close to their breakup velocity and that a magnetic dynamo is a likely the spin-down mechanism.

2 SPIN-DOWN TIME-SCALE AND MAGNETIC WIND

We use as a basis the model of Tout & Pringle (1991, hereinafter TP). In their model mass loss and braking are powered by an $\alpha - \Omega$ dynamo that extracts energy from differential rotation. The differential rotation at the surface and core boundary is maintained by stellar winds and core contraction respectively. Differential rotation throughout the radiative envelope results from redistribution of angular momentum because of the meridional circulation. This can be seen most clearly in models where angular momentum transport is treated as a purely diffusive process. In these models there is very little differential rotation (Potter, Tout & Eldridge 2011). In the case where there is a strong magnetic field, the meridional circulation may be inhibited (Maeder & Meynet 2003) but this conclusion has

been questioned by Maeder & Meynet (2005) who found that, if the meridional circulation is neglected, there is insufficient differential rotation to maintain the magnetic dynamo. In particular, if differential rotation is too strongly suppressed by magnetic fields then it is difficult to explain the existence of slow rotating stars with strong nitrogen enrichment (Hunter et al. 2009; Meynet et al. 2010). Mixing may be driven either by Parker’s instability or by the horizontal rotation-driven turbulence that is itself responsible for the rotationally driven mixing. In TP’s model convection served this purpose. Otherwise our $\alpha - \Omega$ dynamo operates in a similar way. Here we find that the extremely strong horizontal turbulence dominates over the Parker instability, even for low rotation rates.

Our calculations are similar to those of TP. For a star of mass M , radius R_* and angular velocity Ω , the spin-down time-scale τ_{sd} is the total angular momentum divided by the rate of angular momentum loss

$$\tau_{\text{sd}} \approx \frac{k^2 M R_*^2 \Omega}{\dot{J}_{\text{w}}}, \quad (1)$$

where k is the dimensionless stellar radius of gyration ($k^2 \approx 0.1$) and \dot{J}_{w} is the angular momentum loss rate in the wind.

$$\dot{J}_{\text{w}} \approx \dot{M}_{\text{w}} R_{\text{A}}^2 \Omega, \quad (2)$$

where R_{A} is the Alfvén radius (Mestel 1968) where the magnetic energy and kinetic energy in the plasma balance. At R_{A}

$$v_{\text{w}}^2 \approx B^2 / \mu_0 \rho_{\text{w}} \quad (3)$$

where ρ_{w} is the density of wind material and B is the magnetic flux density. We expect the velocity of the wind v_{w} is of the order of the escape velocity so

$$v_{\text{w}}^2 \approx (2GM/R_*). \quad (4)$$

We are imagining a wind launched by magnetic heating of a hot corona and accelerated by coronal pressure here. This differs from the radiatively driven winds normally associated with massive stars that would be somewhat faster. We have also assumed that the wind is not significantly slowed by the star’s gravity. To quantify the effect of changing v_{w} we write

$$v_{\text{w}}^2 \approx \beta(2GM/R_*), \quad (5)$$

where $\beta > 1$ if the wind is accelerated and $\beta < 1$ if it is slowed. Though we expect $R_{\text{A}} > R_*$, if it does fall below R_* then the angular momentum loss rate is instead

$$\dot{J}_{\text{w}} \approx \dot{M}_{\text{w}} R_*^2 \Omega \quad (6)$$

because the material is then lost with the specific angular momentum of the stellar surface.

Outside the star we assume the field strength B depends on radius as

$$B = B_*(R_*/R)^n, \quad (7)$$

where B_* is the field at the surface of the star. The outflow at the Alfvén surface is roughly spherical so that

$$\rho_w \approx \dot{M}_w / (4\pi R_A^2 v_w) \quad (8)$$

and the relation between R_A and R_* follows the equations (3), (7) and (8).

$$\frac{R_A}{R_*} = \left(\frac{4\pi B_*^2 R_*^2}{\mu_0 \dot{M}_w v_w} \right)^{1/(2n-2)}. \quad (9)$$

An undisturbed dipole field has $n = 3$. If the wind were able to open the field lines, so that $n = 2$, then $R_A \geq R_*$ would be larger and the spindown correspondingly faster.

To estimate the wind mass-loss rate \dot{M}_w we consider the rate at which magnetic energy is dissipated and assume that all of this energy L_w goes into launching the wind

$$L_w \approx \alpha \frac{GM\dot{M}_w}{R_*}. \quad (10)$$

We include the factor α because it is likely that not all the magnetic energy released is available to launch the wind. It might simply be radiated away. In that case $\alpha > 1$. On the other hand it might even be possible that less energy is needed to drive the wind because it may be accelerated by radiation driving but only at high metallicity. In such a case $\alpha < 1$.

TP estimated the differential rotation by assuming that convection efficiently redistributes angular momentum. In non-accreting, non-convective, rotating stars the strength of the differential rotation results from the relative strength of meridional circulation and turbulent diffusion. The latter tends to smooth out differential rotation whereas the former in general enhances it. Turbulent diffusion may arise owing to purely hydrodynamic or magnetic instabilities. The exact formulation with and without magnetic fields has been the subject of much discussion and many different models are commonly used (for examples see Heger et al. 2000; Meynet & Maeder 2000; Maeder & Meynet 2004; Meynet et al. 2010). In all cases, the energy for driving the dynamo comes essentially from the excess rotational kinetic energy in the shear, $E_{\text{shear}} = \frac{1}{2}k^2 MR_*^2 \Delta\Omega^2$, where $\Delta\Omega$ approximates the range of angular velocity across the stellar radius. The rate L_+ at which energy is fed into the shear is

$$L_+ \approx \frac{\frac{1}{2}k^2 MR_*^2 \Delta\Omega^2}{\tau_\nu}, \quad (11)$$

where $\tau_\nu \approx \frac{R_*^2}{\nu}$ is the viscous time scale and ν , which behaves as a viscosity, is the radial diffusion coefficient for angular momentum.

We are considering radial variations of the rotation rate under a shellular framework (Zahn 1992) so it is appropriate to take the radial diffusion coefficient for Kelvin-Helmholtz instabilities driven by the shear, referred to as D_{shear} by Meynet & Maeder (2000). Thus $\Delta\Omega \approx \frac{\partial\Omega}{\partial \ln r}|_{r_0}$ is a measure of differential rotation in the star. Because $\frac{\partial\Omega}{\partial \ln r}$ varies throughout the star we choose r_0 to give a typical value within the radiative envelope. We then evaluate the diffusion coefficients at the same radius. For massive stars, the amount of differential rotation is governed by the rate of evolution and the strength of the meridional circulation driven by rotation. Assuming that hydrostatic evolution is slow compared to the advection and diffusion time-scales for angular momentum we may use Zahn (1992)'s model in steady state to get

$$\Delta\Omega \approx \frac{rU\Omega}{5\nu}, \quad (12)$$

where $UP_2(\cos\theta)$ is the radial component of the meridional circulation and $P_2(x)$ is the second Legendre polynomial in x . Using the formulation of Talon & Zahn (1997) for the shear viscosity which dominates the behaviour of the radial differential rotation (Meynet & Maeder 2000) we can approximate this as

$$\Delta\Omega \approx \left(\frac{2rU\Omega N_{\text{T}}^2}{K} \right)^{\frac{1}{3}}, \quad (13)$$

where N_{T}^2 is the Brunt-Väisälä frequency and K is the thermal diffusivity. It has been shown that the stationary approximation to the dynamical equation of Zahn (1992) is quite poor (Meynet & Maeder 2000). However this is in the case where the meridional circulation is inferred from the steady state differential rotation. Here we are using the meridional circulation and turbulent diffusivity calculated from non-stationary models to parameterize the differential rotation so we expect the approximation to be much better than a purely stationary model. We assume that the magnetic field strength does not become strong enough to cause dynamical feedback on either the rotation or the meridional circulation.

Assuming that the energy dissipated from the shear is fed into driving a stellar wind, $L_+ \approx L_{\text{w}}$, we may derive the spin-down time-scales

$$\tau_{\text{sd}} \approx \frac{k^2 M R_*^2 \Omega}{\dot{M}_{\text{w}} [\max(R_*, R_{\text{A}})]^2 \Omega} \approx \begin{cases} \alpha^{1/2} \beta^{1/4} k \left(\frac{2GM}{R_*^3} \right)^{\frac{3}{4}} \Delta\Omega^{-1} \tau_\nu^{\frac{1}{2}} \left(\frac{\mu_0 M}{4\pi B_*^2 R_*} \right)^{\frac{1}{2}} & R_{\text{A}} > R_* \\ \alpha \left(\frac{2GM}{R_*^3} \right) \tau_\nu \Delta\Omega^{-2} & R_{\text{A}} < R_*. \end{cases} \quad (14)$$

The driver of the stellar dynamo is the shear, $\Delta\Omega$, which converts the poloidal field B_{p} to

toroidal B_ϕ . Cyclonic turbulence can then regenerate poloidal flux from toroidal (Parker 1955; Cowling 1981), so we have

$$\frac{dB_\phi}{dt} \approx \Delta\Omega B_p - \frac{B_\phi}{\tau_\phi} \quad (15)$$

and

$$\frac{dB_p}{dt} \approx \left(\frac{\Gamma}{R_*}\right) B_\phi - \frac{B_p}{\tau_p}, \quad (16)$$

where Γ is the dynamo regeneration term and τ_p and τ_ϕ are the time-scales on which poloidal and toroidal flux are lost or destroyed. Following TP we take $\tau_p \approx \tau_\phi \approx 10\tau_A$, where $\tau_A \approx R_*/v_A$ is the Alfvén-wave crossing time in the star for an Alfvén speed v_A . As did TP, we write

$$\Gamma \approx \gamma v_t, \quad (17)$$

where γ is an unknown parameter which describes the efficiency of the regeneration term and v_t is a characteristic turbulent-eddy velocity. The factor γ is very uncertain. Based on the evolution of cataclysmic variables (Warner 1995) TP set it to 10^{-2} for convective stars but found a much lower value of 3×10^{-5} to be sufficient to drive the activity in Ae and Be stars (Tout & Pringle 1994). We use a conservative estimate of $\gamma = 10^{-4}$ here and later show that our estimated spin-down rate is proportional to γ .

In a steady state equations (15) and (16) lead to

$$B_p \approx \frac{\tau_p \Gamma}{R_*} B_\phi \approx 0.1 \frac{v_A}{\Delta\Omega R_*} B_\phi. \quad (18)$$

For each of the models calculated $B_\phi \approx 10^3 B_p$ so that

$$B_\phi \approx v_A \sqrt{\mu_0 \rho_*}, \quad (19)$$

where

$$\rho_* = M/(\mu_0 R_*^3/3) \quad (20)$$

is the mean density of the star. Equation (18) then gives

$$B_p \approx \frac{0.1}{\Delta\Omega} \frac{v_A^2 \sqrt{4\pi\rho_*}}{R_*} \quad (21)$$

and

$$v_A^2 \approx 100 R_* \Gamma \Delta\Omega. \quad (22)$$

We approximate the eddy velocity of turbulence owing to shear by $v_t \approx \max(v_\nu, 0.1v_A)$ where v_ν is the characteristic velocity of rotation-driven eddies. So either the rotation driven

turbulence dominates the dynamo regeneration or, if it is weak enough, the Parker instability sets in. TP set $v_\nu^2 \approx 3D_h\Omega$, where D_h is the horizontal diffusion coefficient. When rotationally driven turbulence dominates $D_h \gg \nu$ and is principally responsible for driving the dynamo. We find this to be the case in all models we consider below and so do not consider the alternative in any detail.

It is the poloidal component of the magnetic field that predominantly drags around the wind so equations (17) and (20) – (22) lead to

$$B_* \approx B_p \approx 10\sqrt{3\mu_0/4\pi\gamma}R_*^{-\frac{3}{2}}M^{\frac{1}{2}}v_t. \quad (23)$$

We define time-scales

$$\tau_\nu = \frac{R_*^2}{\nu}, \quad (24)$$

$$\tau_{\text{shear}} = \Delta\Omega^{-1} \quad (25)$$

and

$$\tau_{\text{dyn}} = \left(\frac{R_*^3}{2GM} \right)^{\frac{1}{2}} \quad (26)$$

so that

$$\dot{M}_w = \frac{\tau_{\text{dyn}}^2}{\tau_{\text{shear}}^2} \frac{Mk^2}{\alpha\tau_\nu} \quad (27)$$

and

$$v_w = \beta^{1/2} \frac{R_*}{\tau_{\text{dyn}}} \quad (28)$$

then with

$$\tau_{\text{turb}} = \frac{R_*}{v_t}. \quad (29)$$

We then have the ratio of the Alfvén to the stellar radius,

$$\frac{R_A}{R_*} \approx \sqrt{10\sqrt{\frac{3\mu_0}{4\pi}} \frac{\gamma^{1/2}\alpha^{1/4}}{k^{1/2}\beta^{1/8}} \left(\frac{\tau_\nu\tau_{\text{shear}}^2}{\tau_{\text{turb}}^2\tau_{\text{dyn}}} \right)^{\frac{1}{4}}}, \quad (30)$$

and a spin-down time-scale,

$$\tau_{\text{sd}} \approx \begin{cases} \frac{\alpha^{1/2}\beta^{1/4}k}{10\gamma\sqrt{3\mu_0/4\pi}} \frac{\tau_{\text{turb}}\tau_{\text{shear}}\tau_\nu^{\frac{1}{2}}}{\tau_{\text{dyn}}^{3/2}} & R_A > R_* \\ \frac{\tau_{\text{shear}}^2\tau_\nu}{\tau_{\text{dyn}}^2} & R_A < R_*. \end{cases} \quad (31)$$

Table 1. Properties of a $20 M_{\odot}$ star spinning with equatorial velocity v (first column). The mass loss rate $\dot{M}/M_{\odot} \text{ yr}^{-1}$ is the predicted mass loss rate driven by the magnetic wind. The second column lists the ratio of the Alfvén radius to the stellar radius. This ratio is proportional to $\alpha^{1/4}\beta^{-1/8}\gamma^{1/2}$. The star’s surface magnetic field strength is B_* and typical radial and horizontal diffusion coefficients are ν and D_h . Then $\Delta\Omega$ is an estimate of the differential rotation through the star, τ_{ms} is its main-sequence lifetime, were it to continue to conserve its angular momentum, and τ_{sd} is its spin-down time-scale which is proportional to $\alpha^{1/2}\beta^{1/4}\gamma^{-1}$.

$v/\text{km s}^{-1}$	R_A/R_*	B_*/G	$\nu/\text{cm}^2 \text{ s}^{-1}$	$D_h/\text{cm}^2 \text{ s}^{-1}$	$\dot{M}/M_{\odot} \text{ yr}^{-1}$	$\Delta\Omega/\text{s}^{-1}$	$\tau_{\text{ms}}/\text{yr}$	$\tau_{\text{sd}}/\text{yr}$
50	2.1	14	1.0×10^8	4.5×10^{12}	2.7×10^{-10}	2.3×10^{-5}	7.8×10^6	1.7×10^9
100	1.7	37	3.3×10^8	1.4×10^{13}	3.9×10^{-9}	4.8×10^{-5}	7.9×10^6	1.8×10^8
150	1.5	62	7.0×10^8	2.5×10^{13}	1.9×10^{-8}	7.4×10^{-5}	8.1×10^6	4.6×10^7
200	1.4	97	1.0×10^9	4.2×10^{13}	5.3×10^{-8}	9.9×10^{-5}	8.3×10^6	1.8×10^7
300	1.3	160	2.0×10^9	6.6×10^{13}	2.2×10^{-7}	1.5×10^{-4}	8.9×10^6	5.6×10^6
400	1.2	220	2.6×10^9	9.4×10^{13}	5.1×10^{-7}	2.0×10^{-4}	9.4×10^6	2.7×10^6
500	1.2	281	3.0×10^9	1.2×10^{14}	9.0×10^{-7}	2.5×10^{-4}	9.7×10^6	1.6×10^6

3 APPLICATION TO ROTATING MASSIVE STARS

To investigate the spin down of a $20 M_{\odot}$ star we evaluated the spin-down time-scale (equation 31), with $\alpha = 1$, $\beta = 1$ and $\gamma = 10^{-4}$, for a series of surface rotation rates. We used the Cambridge STARS code (Eggleton 1971), which has previously been updated by many authors (most importantly by Pols et al 1995; Stancliffe & Eldridge 2009). The version we have used has been modified to include the effects of rotation by Potter, Tout & Eldridge (2011). With this we can estimate U , D_h and ν for the star on the main sequence. In this case the Alfvén radius is always larger than the radius of the star. We chose to estimate the turbulence at r_0 where $\nabla - \nabla_{\text{ad}}$ has a minimum. In all cases $r_0 \approx 4 R_{\odot}$ from the centre of the star which has a total radius $R_* \approx 6 R_{\odot}$. In Table 1 we list various properties of the models, including the spin-down time-scale. We note that $R_A/R_* \propto \alpha^{1/4}\beta^{-1/8}\gamma^{1/2}$ and $\tau_{\text{sd}} \propto \alpha^{1/2}\beta^{1/4}\gamma^{-1}$. The mass-loss rate driven by this mechanism is at a rate of $\dot{M}_w \approx \alpha^{-1} 10^{-6} M_{\odot} \text{ yr}^{-1}$ for the most rapid rotators and escapes at $v_w \approx \beta^{1/2} 10^3 \text{ km s}^{-1}$. This is a slower, but not much slower, wind than that typically driven by radiation, at about $3,000 \text{ km s}^{-1}$. So we may expect $\beta \leq 3$ and this to have little effect. Also there is no a priori reason why such a magnetically driven wind, or its consequences, should depend on metallicity. The spin-down time-scale becomes shorter than the main-sequence lifetime for stars initially rotating faster than around 200 km s^{-1} . It is much shorter for rapid rotators and so a star cannot continue to rotate at a high velocity throughout its entire main-sequence lifetime. It appears unlikely that homogeneous evolution, induced by very rapid rotation, can last long enough to change the fate of the stars even when they are of low metallicity. We note that ζ Pup, a $53.9 M_{\odot}$ star (Repolust, Puls & Herrero 2004) with an observed rotational velocity of $v \sin i \approx 200 \text{ km s}^{-1}$ (Kudritzki, Simon & Hamann 1983), has an observed mass loss rate of

about $\dot{M}_w \approx 3.5 \times 10^{-6} M_\odot \text{yr}^{-1}$ (Cohen et al. 2010). This mass loss rate is not inconsistent with our estimate.

3.1 Uncertainty in the estimation

We have assumed perfect efficiency for the transfer of energy from shear to the magnetically driven wind. This is unlikely to be the case and so the spin-down time-scales are likely to be larger than those predicted. However, if the efficiency factor α does not depend on the rotation rate, the prediction that rapid rotators spin down on a much shorter time-scale is still valid. Indeed, unless the efficiency is extremely low ($\alpha \geq 30$), the spin-down time-scale for rapid rotators is much shorter than their main-sequence lifetime.

There is also uncertainty introduced by our choice of r_0 . We have chosen it by a standardised method within the radiative envelope. However, if we vary r_0 for a $20 M_\odot$ star with surface rotation of 200 km s^{-1} , the spin-down time-scale varies between 7.2×10^4 and $2.1 \times 10^7 \text{ yr}$. It is unlikely to be close to the lower limit because this is the spin-down time-scale when r_0 is taken close to the convective core. This is only representative of a small part of the radiative envelope and energy deposited there is unlikely to be very influential in driving the stellar wind.

A major uncertainty is γ , the standard dynamo regeneration term. In our calculation, we have used a conservative value of 10^{-4} but it could be larger by as much as two orders of magnitude. If that were the case then, the spin-down time-scales ($\tau_{\text{sd}} \propto \gamma^{-1}$) would be much shorter. Given the uncertainties in our estimate, we cannot draw a quantitative conclusion for a star with a moderate spin rate. However, our results do suggest that stars rotating near their break-up velocity, or stars with very efficient rotationally driven mixing, are likely to spin down very quickly by driving a magnetically locked wind. This effect cannot be ignored.

As table 1 shows, $R_A > R_*$ in all of the cases examined here. This means that $\tau_{\text{sd}} \propto \Delta\Omega$, assuming that the turbulent viscosity is unaffected. If the differential rotation is strongly suppressed by the presence of a magnetic field then it may extend the spindown timescale of the star. However, the degree to which this happens is extremely uncertain. In particular, if solid body rotation is enforced it is difficult to reconcile models and observations (Meynet et al. 2010).

4 CONCLUSION

We have estimated the spin-down time-scale for rapidly rotating non-convective stars. With a conservative value for the magnetic field regeneration term γ , the spin-down time-scale is likely to be long for slowly rotating stars for which the horizontal turbulence driven by rotation is weaker. However, for more rapid rotators, the spin-down time-scale can be much shorter than the main-sequence lifetime. Given the uncertainties in the coefficient of mixing, viscosity and the appropriate mixing length, more detailed models of magnetic spin down driven by rotational mixing must be developed before we can decide on whether rapid spin can lead to homogeneous mixing and on the importance of rotationally driven mixing at lower spin rates. These calculations do however mean that we should not automatically assume that even very low-metallicity stars can continue to spin rapidly throughout their main-sequence lives.

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